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NONLINEAR SYSTEMS

Global Stability of a Second-Order Affine Switching System

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Abstract—Stability of an affine switching system is studied. The system comes to existence when stabilizing a chain of two integrators by means of a feedback in the form of nested saturators. The use of such a feedback allows one to easily take into account boundedness of the control resource, to constrain the maximum velocity of approaching the equilibrium state, which is especially important in the case of large initial deviations, and to ensure desired characteristics of the transient process, such as a given exponential rate of the deviation decrease near the equilibrium state. It is proved that the closed-loop system is globally stable.

Keywords: stabilizing a chain of two integrators, affine switching system, global stability, nested saturators

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1. INTRODUCTION

Hybrid systems are dynamical systems that exhibit both continuous-time and discrete-time behavior; i.e., systems whose states vary continuously but may also jump [1]. A switching system is a hybrid dynamical system consisting of a number of subsystems and a switching law determining which subsystem is active at a current moment of time [2]. Systems of this kind are encountered in many control problems in various fields of science and technology [1, 2]. One of the most important problems in study of switching systems is that of stability [2–4]. It is stability of the switching system under consideration that is discussed in this work.

The affine switching system under study comes to existence when stabilizing a chain of two integrators by means of a feedback in the form of nested saturators. The problem of stabilizing chains of integrators was widely discussed in the literature during last several decades (see, e.g., [5–7] and references therein). The interest to this problem is motivated by the fact that many real-life systems in applications (e.g., mechanical planar ones) are modeled by chains of integrators; moreover, controls developed for chains of integrators can be easily extended to larger classes of systems.

Feedbacks in the form of nested saturators were studied and used for stabilizing integrators in many publications (see, e.g., [5, 6, 8–11] and references therein). However, the author is not aware of the works the results of which could be used for establishing stability of the system closed by the feedback considered in the paper. The general case of the nth-order integrator was discussed, for example, in [5, 6]. However, global stability of the system closed by a feedback in the form of n nested saturators was proved only for the special case where the limits of the saturation functions satisfy certain inequalities [5, Theorem 2.1], which are not fulfilled for the feedback used in this study. Global stability of the second-order integrator stabilized by a feedback in the form of nested saturators, but with the reversed order of the arguments (see the next section for more detail), was proved in [8, 9]. However, the approach employed in these works is not applicable to the case of the feedback considered in this paper.

The feedback of the form considered in the paper was studied earlier in some works. For example, in [10, 11], optimization problems of selecting the feedback coefficients were discussed; in [12–14], it was used in the synthesis of controllers for stabilizing higher-order integrators. In these works, stability of the system closed by such a feedback was implied but was not proved. The goal of this study is to prove global stability of the system discussed filling thus this gap. The interest in the feedback in the form of nested saturators is explained by a number of remarkable features of the closed-loop system obtained, which is discussed in the next section.

2. PROBLEM STATEMENT

Consider the problem of stabilizing a chain of two integrators:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = U(x), \tag{1}$$

where $x \equiv [x_1, x_2]^{\mathrm{T}}$, by means of a continuous feedback with a constrained control resource $U_{\max} = k_4 : U(x) = -k_3(x_2 + k_2\mathrm{sat}(k_1x_1))$ for $|U(x)| \leqslant k_4$ and $U(x) = -k_4\mathrm{sign}(k_3(x_2 + k_2\mathrm{sat}(k_1x_1)))$ for $|U(x)| > k_4$. In the compact form, control $U(x)$ is written as

$$U(x) = -k_4\mathrm{sat}\left(\frac{k_3}{k_4}(x_2 + k_2\mathrm{sat}(k_1x_1))\right), \tag{2}$$

where $\mathrm{sat}(\cdot)$ is the nonsmooth saturation function: $\mathrm{sat}(w) = w$ for $|w| \leqslant 1$ and $\mathrm{sat}(w) = \mathrm{sign}(w)$ for $|w| > 1$. The advantage of the feedback in the form of nested saturators is that not only the control constraint is automatically satisfied but also the maximum speed of approaching the equilibrium is limited: if we set $k_2 = V_{\max}$, then, for any initial deviation, $\dot{x}_1(t) \leqslant V_{\max}$ as long as $x_2(0) \leqslant V_{\max}$ [11]. Moreover, any desired type of the equilibrium (node, pole, or center) and any desired value of the exponential rate of deviation decrease near the origin can be ensured by appropriate choice of the coefficients k_1 and k_3 [11].

As noted in the Introduction, in [8, 9], a feedback of form (2) with the reverse compared to (2) order of arguments, where the argument of the internal saturator is velocity x_2 and that of the external saturator is deviation x_1, was considered (in [9], the argument of the external saturator depends additionally on the velocity squared x_2^2), and it was proved that the second-order integrator closed by such a feedback is globally stable. The proof in both works is based on the existence of a Lyapunov function in the form of the sum of a quadratic and integral terms. For system (1), (2), however, this expedient is not applicable, since no Lyapunov function is available.

2.1. Equivalent Representation in the Form of an Affine Switching System

Let us first show that system (1), (2) is an affine switching one. Consider partitioning of plane (x_1, x_2) into five sets (Fig. 1). In the set D_1, we include all points where both saturators are not saturated:

$$D_1 = \{(x_1, x_2) : |x_1| < 1/k_1,\ |x_2 + k_1x_1| < k_4/k_3\}$$

(the inclined strip bounded by the dashed lines in Fig. 1). The set D_2 consists of all points where the internal saturator reaches saturation, while the external one does not:

$$D_2 = \{(x_1, x_2) : |x_1| \geqslant 1/k_1,\ |x_2 + k_2\mathrm{sgn}(x_1)| < k_4/k_3\}.$$

As can be seen from the figure, D_2 consists of the two disjoint sets D_2^- and D_2^+ belonging to the left and right half-planes, respectively (two disjoint horizontal strips in Fig. 1). The set

$$D_3 = \{(x_1, x_2) : |x_2 + k_2\mathrm{sat}(k_1x_1))| > k_4/k_3\}$$

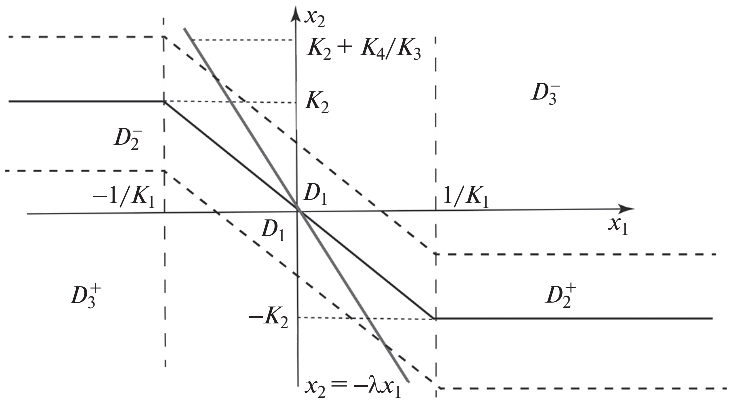

Fig. 1.

includes all points where the external saturator reaches saturation. Like D_2, D_3 consists of two nonintersecting sets D_3^- and D_3^+ lying above and below the line $x_2 = -\text{sat}(k_1x_1)$ (the solid broken line in Fig. 1), in which $U_1(x) \equiv -k_4$ and $U_1(x) \equiv +k_4$, respectively.

From formula (2), it can be seen that $U(x)$ is a piecewise continuous function:

$$U(x) = \begin{cases} -k_3x_2 - k_1k_2k_3x_1, & (x_1, x_2) \in D_1, \\ -k_3(x_2 - k_2), & (x_1, x_2) \in D_2^-, \\ -k_3(x_2 + k_2), & (x_1, x_2) \in D_2^+, \\ -k_4, & (x_1, x_2) \in D_3^-, \\ +k_4, & (x_1, x_2) \in D_3^+, \end{cases} \tag{3}$$

and the closed-loop system (1), (2) includes five linear systems, with the switching between them being state dependent according to equation (3). The goal of this study is to prove global stability of this system.

The standard method of proving stability of linear switching systems—determining a common Lyapunov function for all systems—is not applicable in this case, since the origin is an equilibrium point for only the first system with the domain D_1. The other four systems, although linear ones, have no equilibria at all; i.e., we deal with an *affine switching system.* The standard method of proving stability of general-form nonlinear systems with the help of a Lyapunov function (like, e.g., in [9]) cannot be applied either since we failed to find one for the system under consideration.

2.2. Representation in Dimensionless Form

To begin with, we simplify the task by reducing the number of the system parameters. Clearly, stability of the system does not depend on particular values of the control resource k_4 and maximum velocity k_2, so that we can set them equal to one. Indeed, turning to dimensionless variables $\tilde{x}_1 = k_4x_1/k_2^2$, $\tilde{x}_2 = x_2/k_2$ and time $\tilde{t} = k_4t/k_2$, we reduce system (1), (2) to the form

$$\frac{d\tilde{x}_1}{d\tilde{t}} = \tilde{x}_2, \quad \frac{d\tilde{x}_2}{d\tilde{t}} = -\text{sat}(\tilde{k}_3(\tilde{x}_2 + \text{sat}(\tilde{k}_1\tilde{x}_1))), \tag{4}$$

where $\tilde{k}_1 = k_1k_2^2/k_4$ and $\tilde{k}_3 = k_2k_3/k_4$, with unitary dimensionless control resource $\tilde{k}_4 = 1$ and unitary maximum velocity $\tilde{k}_2 = 1$. In what follows, all variables and constants are assumed dimensionless and are denoted by the same symbols (without tilde) as dimensional ones. As before, we use the dot notation to denote the derivatives with respect to the dimensionless time. Moreover, without loss of generality, we will select coefficients k_1 and k_3 from a one-parameter family parameterized by the exponential rate λ of the deviation decrease near the origin:

$$k_1 = \lambda/2, \quad k_3 = 2\lambda, \ \lambda > 0. \tag{5}$$

With these coefficients, system (1) closed by the feedback (2) takes the form

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -\mathrm{sat}(2\lambda(x_2 + \mathrm{sat}(\lambda x_1/2))). \tag{6}$$

In D_1, we have linear system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -\lambda^2 x_1 - 2\lambda x_2, \tag{7}$$

the characteristic equation of which has two identical roots $\lambda_1 = \lambda_2 = -\lambda$; i.e., the origin is a stable degenerate node. In other domains, we have the following systems:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -2\lambda(x_2 - 1), \quad (x_1, x_2) \in D_2^-, \tag{8}$$
$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -2\lambda(x_2 + 1), \quad (x_1, x_2) \in D_2^+, \tag{9}$$
$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -1, \quad (x_1, x_2) \in D_3^-, \tag{10}$$
$$\dot{x}_1 = x_2, \quad \dot{x}_2 = 1, \quad (x_1, x_2) \in D_3^+. \tag{11}$$

Equation (6) is an equivalent representation of the switching system (7)–(11).

3. PROOF OF GLOBAL STABILITY

First, we prove that, in the study of stability of the system. we can confine ourselves to the consideration of the trajectories beginning in the set D_1.

Proposition. System (7)–(11) is globally asymptotically stable if and only if any trajectory beginning in the set D_1 asymptotically tends to the origin.

The necessity of the assertion is evident. The sufficiency is proved by showing that any trajectory with arbitrary initial conditions occurs in the set D_1 in a finite time. Let us prove this. Indeed, from equations (10) and (11), it is seen that trajectories of the system in D_3^- and D_3^+ are parabolas

$$x_1 = \mp\frac{1}{2}x_2^2 + C. \tag{12}$$

Since any parabola cannot lie entirely in D_3^- or D_3^+ (see Fig. 1) and the system moves with the constant acceleration, it inevitably occurs in a finite time in either D_1 or D_2. Further, from equations (8) and (9), it is seen that, in D_2^- (D_2^+), the system moves in the positive (negative) direction of x_1 and $x_2(t) \to 1$ ($x_2(t) \to -1$). Then, it follows that the system inevitably enters D_1 in a finite time. Thus, for any initial conditions, after at most two switchings, the system occurs in the set D_1. Further, only trajectories beginning in D_1 are considered.

Theorem 1. *System* (6) *is globally asymptotically stable for any* $\lambda > 0$.

Proof. Let us find out whether the system can enter D_2 from D_1. For definiteness, consider the boundary between D_1 and D_2^+. From the first equation in (9), it is seen that the trajectory can intersect the boundary only if x_2 is positive, i.e., when the half-width $1/2\lambda$ of the strip D_2 is greater than one, like in the case shown in Fig. 2, which takes place only when $\lambda < 1/2$. Since the right-hand side of the second equation in (9) in this case is negative, $x_2(t)$ will change sign in a finite time, This, in turn, will change the direction of motion along x_1-axis, bringing thus the system to D_1 again. Note that the segment of the asymptote $x_2 = -\lambda x_1$ (the bold line in Fig. 2) of the linear system (7) for which $|x_1| \leqslant 2/\lambda$ lies completely in D_1. Since no trajectory of the system can intersect the asymptote in D_1, all trajectories asymptotically tend to the origin. The case of

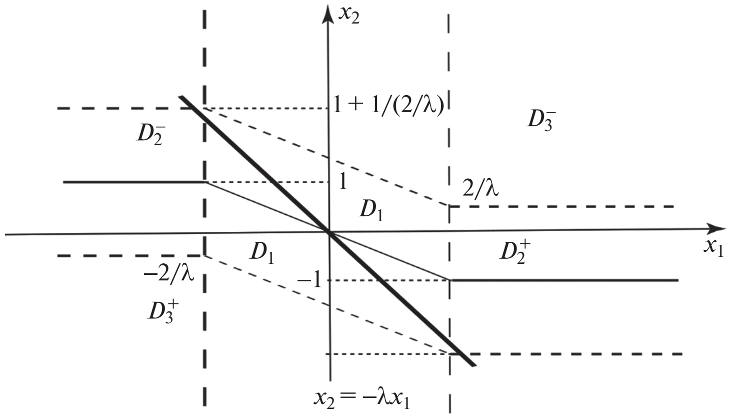

Fig. 2.

negative x_1 is considered similarly. Thus, for small $\lambda < 1/2$, the system is globally stable. Note that, in this case, any trajectory beginning in the set D_1 can intersect the boundary between the sets (D_1 and D_2) not more than twice.

Let us determine conditions under which the system can switch from D_1 to D_3. The boundary between the sets (dashed lines in Fig. 1) is given by the equations

$$x_2 = -\frac{\lambda}{2}x_1 \pm \frac{1}{2\lambda}, \quad -\frac{2}{\lambda} \leqslant x_1 \leqslant \frac{2}{\lambda}, \tag{13}$$

where the plus sign before the second addend corresponds to the upper boundary (the boundary between D_1 and D_3^-), and the minus sign, to the lower boundary. A trajectory can intersect the boundary only if its slope is less than that of the boundary, which is $\lambda/2$. From equations (10) and (11), we find that the slope of the trajectory on the boundary is $1/x_2$, from which it follows that the trajectory can intersect the upper (lower) boundary only at the points with ordinates satisfying the inequality $x_2 > 2/\lambda$ ($x_2 < -2/\lambda$), i.e., in the region

$$|x_2| > 2/\lambda. \tag{14}$$

Since the maximum value of $|x_2|$ in D_1 is achieved in two angular points with ordinates $\pm(1 + 1/(2\lambda))$, trajectories cannot intersect the boundary when $\lambda \leqslant 3/2$.

Thus, global stability of the system is proved for all $\lambda \leqslant 3/2$. Moreover, we proved the following nontrivial assertion.

Lemma 1. *Let $1/2 \leqslant \lambda \leqslant 3/2$. Then, D_1 is an invariant set of the switching system* (6).

Note that D_1 in this case is an invariant set of the linear system (7) either. Now, let us prove that the system is globally stable for any greater values of λ. From the above calculations, it follows that, for $\lambda > 3/2$, the system can pass from D_1 to only D_3. Consider, for definiteness, the upper part of the phase plane, where $U(x) < 0$. Constant C on the right-hand side of (12) depends on the coordinates of the point where the system passes from D_1 to D_3^-. Let x_{2*} denote the ordinate of the point where the trajectory intersects the boundary (the abscissa is uniquely determined from the equation of the boundary (13)). Then,

$$C \equiv C(x_{2*}) = \frac{1}{2}\left(x_{2*}^2 - \frac{4x_{2*}}{\lambda} + \frac{2}{\lambda^2}\right).$$

Substituting the right-hand side of this formula for C in (12) and solving the quadratic equation obtained, we find the ordinate (denote it as x_{2**}) of the second intersection point of the parabola and the boundary (13), where the system switches from (10) to (7):

$$x_{2**} = \frac{4}{\lambda} - x_{2*}. \tag{15}$$

With regard to the inequalities

$$\frac{2}{\lambda} < x_{2*} \leqslant 1 + \frac{1}{2\lambda},$$

it follows from (15) that x_{2**} satisfies the inequalities

$$\frac{2}{\lambda} > x_{2**} \geqslant -1 + \frac{7}{2\lambda} > -1 + \frac{1}{2\lambda};$$

i.e., the second intersection point of the parabola and the line (13) belongs to the boundary between D_3^- and D_1, and, hence, the trajectory passing from D_1 to D_3 cannot occur in D_2. Thus, when $\lambda > 3/2$, switchings are possible only between the three systems with the domains D_1, D_3^-, and D_3^+. Similarly, two successive points of intersection of the boundry between D_1 and D_3^+ are found to be

$$x_{2**} = -\frac{4}{\lambda} - x_{2*}, \quad -\frac{2}{\lambda} < x_{2*} \leqslant -1 - \frac{1}{2\lambda}. \tag{16}$$

Since any trajectory cannot have self-intersections and does not go to infinity, it will suffice to prove that no closed trajectory (cycle) exists [15]. Let us assume the contrary: suppose that there exists a closed trajectory. From the above discussions, it follows that such a trajectory consists of four segments: two segments in D_1, one segment in D_3^+, and one segment in D_3^-, with the motion along the trajectory being clockwise.

Let us show that there exists a positive definite function that decreases on all segments of the cycle, from which it follows that the trajectory cannot be closed. Note that we do not mean a Lyapunov function, since we do not require negativeness of its derivative by virtue of system (6) at all points of the trajectory. We seek for a function the total variation of which ater passing the entire segment completely lying in one of the regions D_1, D_3^-, or D_3^+ is negative. For a candidate of the desired function, we take a quadratic Lyapunov function $F = \lambda^3 x^{\mathrm{T}} P x$ (the multiplier λ^3 is introduced for the convenience of notation) of the linear system (7). Here, P is a positive definite matrix of order two satisfying the linear matrix inequality (LMI) $A^{\mathrm{T}}P + PA < 0$ [16], and A is the matrix of the linear system (7):

$$A = \begin{pmatrix} 0 & 1 \\ -\lambda^2 & -2\lambda \end{pmatrix}.$$

In [17], it was shown that matrix P can be represented in the form

$$P = \begin{pmatrix} \lambda & q_1/2 \\ q_1/2 & q_2/\lambda \end{pmatrix}, \tag{17}$$

where $q_1, q_2 > 0$ belong to the ellipse Ω (Fig. 3) defined by the inequality

$$(q_2 - q_1 - 1)^2 + (q_1 - 2)^2 \leqslant 4. \tag{18}$$

Let us find out whether there exist $(q_1, q_2) \in \Omega$ such that function F decreases on each of the four segments.

The derivative of function F by virtue of system (7) is negative by definition, which guarantees that function F decreases on two trajectory segments lying in D_1. On the segments lying in D_3, negativeness of the derivative of F is not guaranteed; however, we prove further that the integral variation of F on each of these segments is negative; i.e., the value of the function at the boundary

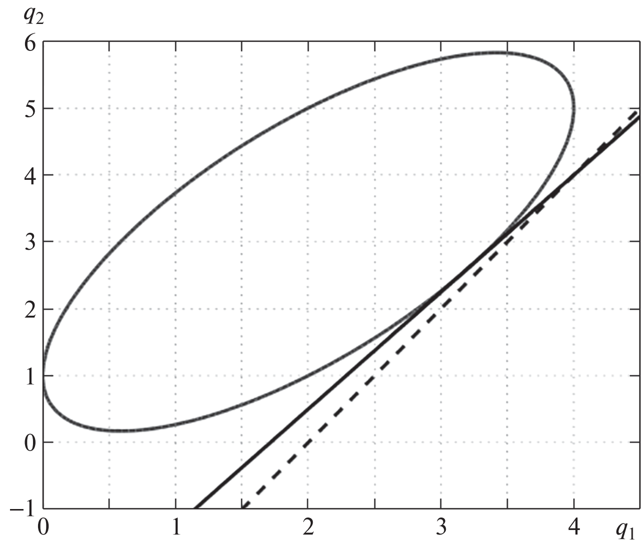

Fig. 3.

point where the system passes from D_1 to D_3 is greater than that at the point where it returns from D_3 to D_1.

Substituting the right-hand side of (17) into F, we get $F(x) = \lambda^2(\lambda^2 x_1^2 + \lambda q_1 x_1 x_2 + q_2 x_2^2)$. Expressing x_1 in terms of x_2 from the equation of boundary (13) and substituting it into the right-hand side of the formula for function F, we obtain the value of F on the upper boundary of D_1:

$$F(x_2) = c_1 x_2^2 - c_2 x_2 + 1, \tag{19}$$

where $c_1 = \lambda^2(q_2 - 2q_1 + 4)$ and $c_2 = \lambda(4 - q_1)$. It follows from inequality (18) that $q_1 < 4$ and, hence, $c_2 > 0 \ \forall q_1, q_2 \in \Omega$. It is easy to show that ellipse (18) has no intersections with the straight line $q_2 - 2q_1 - 4 = 0$ (the dashed line in Fig. 3) and lies above it; hence, $c_1 > 0 \ \forall q_1, q_2 \in \Omega$. Let us find the variation ΔF of function F on the trajectory segment lying in D_3^-. With regard to (19) and (15), at the beginning and end points of the segment, the function takes values

$$F(x_{2*}) = c_1 x_{2*}^2 - c_2 x_{2*} + 1, \quad F(x_{2**}) = c_1(4/\lambda - x_{2*})^2 - c_2(4/\lambda - x_{2*}) + 1.$$

Then, it follows that

$$\begin{aligned}\Delta F = F(x_{2**}) - F(x_{2*}) &= c_1(16/\lambda^2 - 8x_{2*}/\lambda) + 2c_2 x_{2*} - 4c_2/\lambda \\ &= (2c_2 - 8c_1/\lambda)x_{2*} + 16c_1/\lambda^2 - 4c_2/\lambda = -(8q_2 - 14q_1 + 24)(\lambda x_{2*} - 2).\end{aligned}$$

It is easy to verify that the straight line $8q_2 - 14q_1 + 24 = 0$ (solid line in Fig. 3) touches ellipse Ω and lies below it, so that the first multiplier is positive. Since, according to (14), $x_{2*} > 2/\lambda$, the second multiplier is positive either, so that $\Delta F < 0$ for any $(q_1, q_2) \in \Omega$.

Repeating these calculations for the lower boundary of set D_1 and taking into account (16), we find

$$F(x_2) = c_1 x_2^2 + c_2 x_2 + 1 \tag{20}$$

and

$$\begin{aligned}\Delta F &= c_1(4/\lambda + x_{2*})^2 - c_2(4/\lambda + x_{2*}) - c_1 x_{2*}^2 - c_2 x_{2*} \\ &= (8q_2 - 14q_1 + 24)(\lambda x_{2*} + 2).\end{aligned} \tag{21}$$

According to (14), on the lower boundary, $x_{2*} < -2/\lambda$, the second multiplier in (21) is negative; hence, the variation of function F on the trajectory segment lying in D_3^+ is also negative. Thus, for any $(q_1, q_2) \in \Omega$, the value of the quadratic Lyapunov function of the linear system (7) decreases after passing each trajectory segment, from which it follows that the trajectory cannot be a closed curve. The theorem is proved.

Numerical examples illustrating behavior of the trajectories of integrator (1) stabilized by means of feedback (2) can be found in [10, 11].

4. CONCLUSIONS

The problem of stabilizing a second-order affine system consisting of five subsystems, of which only one has a stable equilibrium, with a state-depending switching law has been considered. The system under study comes to existence when applying a feedback in the form of nested saturators for stabilizing a chain of two integrators. The advantages of the considered feedback are its continuity and boundedness, as well as the possibility to ensure desired characteristics of the transient process. By means of an appropriate selection of the four feedback coefficients, it is easy to ensure a desired type of the equilibrium and a desired exponential rate of the deviation decrease near the equilibrium state, as well as to constrain the maximum speed of approaching the equilibrium state, which is especially important in the case of large initial deviations. The main result of the study is the proof of global stability of the considered affine switching system.

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