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**NOTES, CHRONICLES, INFORMATION**

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Dykhta Vladimir Aleksandrovich  
(1949–2025)

On April 2, 2025, most regrettably, Vladimir Alexandrovich Dykhta, a remarkable Russian and Soviet mathematician, an Honored Scientist of the Russian Federation (1997), a remarkable person, Doctor of Physical and Mathematical Sciences, Professor, passed away. He worked until the end of his life in the positions of Chief Researcher at the Matrosov Institute for System Dynamics and Control Theory, Siberian Branch of Russian Academy of Sciences (ISDCT SB RAS)<sup>1</sup> and Professor of the Department of Computational Mathematics and Optimization at the Institute of Mathematics and Information Technology, Irkutsk State University (IMIT ISU)<sup>2</sup>. On the websites of ISDCT SB RAS, ISU, and IMIT ISU, the corresponding obituaries are posted<sup>3,4,5</sup>. Vladimir Alexandrovich has made an outstanding contribution to the theory of optimal control and its applications. The given below list of the publications [1–92] does not pretend to be complete. In these publications, V.A. Dykhta is the single author or a co-author. The articles were published in the journals “Automation and Remote Control” (“Avtomatika i Telemekhanika”), “Doklady Mathematics” (“Doklady Akademii Nauk”), “Engineering Cybernetics” (“Izvestija Akademii Nauk SSSR. Tehnicheskaja Kibernetika”), “Journal of Computer and System Sciences International” (“Izvestija RAN. Teorija i Sistemy Upravlenija”), “Mathematical Notes of the Academy of Sciences of the USSR” (“Matematicheskie Zametki”), “Differentsial’nye Uravneniya”, “Proceedings of the Steklov Institute of Mathematics” (“Trudy Matematicheskogo Instituta imeni V.A. Steklova”), “Compu-

<sup>1</sup> <http://idstu.irk.ru/en>

<sup>2</sup> <https://math.isu.ru/>

<sup>3</sup> <http://idstu.irk.ru/en/inews/ushel-iz-zhizni-zasluzhenny-deyatel-nauki-rf-vladimir-dykhta> (the photo is from this web page).

<sup>4</sup> <https://isu.ru/ru/news/2025/details/news-id2025necrologDYCHTA>

<sup>5</sup> <https://math.isu.ru/export/sites/math/ru/media/announces/2025/.galleries/docs/necrolog.pdf>

tational Mathematics and Mathematical Physics” (“Zhurnal Vychislitel’noj Matematiki i Matematicheskoy Fiziki”), “Soviet Mathematics (Izvestiya VUZ. Matematika)” and “Russian Mathematics” (“Izvestija Vysshih Uchebnyh Zavedenij. Matematika”), “Journal of Mathematical Sciences” (“Itogi Nauki i Tehniki. Sovremennaja Matematika i ee Prilozhenija. Tematicheskie Obzory”), “Proceedings of Krasovskii Institute of Mathematics and Mechanics UB RAS” (“Trudy Instituta Matematiki i Mekhaniki UrO RAN”), “Siberian Mathematical Journal” (“Sibirskij Matematicheskij Zhurnal”), “Bulletin of Irkutsk State University. Series Mathematics” (“Izvestija Irkutskogo Gosudarstvennogo Universiteta. Serija Matematika”), “Buryat State University Bulletin. Mathematics, Informatics” (“Vestnik Burjatskogo Gosudarstvennogo Universiteta. Matematika, Informatika”), “Journal of Optimization Theory and Applications”, “European Journal of Control”, etc. V.A. Dykhta is a co-author of the monographs’ series [5–15]. In particular, [14] is the second edition of the monograph “Optimal Impulse Control with Applications” by V.A. Dykhta, O.N. Samsonyuk.

V.A. Dykhta was born on October 1, 1949 in Irkutsk. He graduated from the Faculty of Mathematics, Irkutsk University in 1972 and worked at his Alma Mater for decades in various positions: assistant, senior lecturer, associate professor, professor, and the head of a department. Under the supervision of Vladimir Iosifovich Gurman<sup>6</sup>, a well-known Russian and Soviet scientist and specialist in the theory of optimal control and its applications, V.A. Dykhta prepared the candidate’s dissertation “Dostatochnye uslovija optimal’nosti osobyh rezhimov” (“Sufficient Conditions for Optimality of Singular Regimes”) in Irkutsk and defended this dissertation in 1979 in Sverdlovsk [1, 2]. In 1992, the doctoral dissertation “Rasshirenie zadach optimal’nogo upravlenija i variacionnyj princip maksimuma” (“Extension of Optimal Control Problems and the Variational Maximum Principle”)<sup>7</sup> prepared by Vladimir Aleksandrovich during his doctoral studies at the Irkutsk Computing Center, Siberian Branch of the USSR Academy of Sciences was defended by Vladimir Aleksandrovich at the Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences (Novosibirsk) [3, 4]. In 1992–2007, Vladimir Alexandrovich was the head of the Department of Higher Mathematics at the Irkutsk State Academy of Economics (now Baikal State University). Since 2008, the main place of the work for Vladimir Alexandrovich was ISDCT SB RAS, where he headed the Laboratory of Optimal Control, a department and was a Chief Researcher. V.A. Dykhta was a member of some dissertation councils, the organizing committees’ head of several international school-seminars “Nonlinear Analysis and Extremal Problems” (NLA), including, for example, the first school-seminar (2008)<sup>8</sup>. We also note the work of V.A. Dykhta as a supervisor and a lecturer, who has prepared 11 candidates of sciences (for example, N.V. Derenko (1994), O.N. Samsonyuk (1999), N.V. Antipina (2003), and S.P. Sorokin (2012)) and 2 doctors of sciences, has published a number of textbooks, including [16–21], has given a large number of university courses, mainly in Irkutsk, and also as a visiting professor in Ulan-Ude (in the Buryat State University and East Siberian State Technological University).

An important component of the scientific heritage of V.A. Dykhta is a development of the fundamental results of V.I. Gurman and V.F. Krotov<sup>9</sup>. One of the key characteristics of the scientific work of Vladimir Aleksandrovich is his holistic vision of the subject of the various necessary and sufficient conditions for optimality, taking into account subtle theoretical aspects. V.A. Dykhta’s

<sup>6</sup> About V.I. Gurman (1934–2016), there are the articles [80, 81], [90th Anniversary of the Birth Vladimir Iosifovich Gurman // Autom. Remote Control, 2024, pp. 1128–1130.]

<sup>7</sup> The scientific advisor was Doctor of Physical and Mathematical Sciences, Professor Aleksandr Aleksandrovich Tolstonogov. He is an Honored Scientist of the Russian Federation since 2002 and a Corresponding Member of the Russian Academy of Sciences since 2006 (<http://idstu.irk.ru/en/aatol>). At ISDCT SB RAS, he is an Advisor to the Director, head of a department, and a Chief Researcher.

<sup>8</sup> <http://idstu.irk.ru/en/node/93/95>

<sup>9</sup> About V.F. Krotov (1932–2015), a well-known Russian and Soviet scientist, specialist in the theory of optimal control theory and its applications, there are the articles [Khrustalev M.M., *AiT*, 2022, No. 5, pp. 164–168 (in Russian)], [“Krotov Vadim Fedorovich”, <https://www.ipu.ru/en/node/71453>].

scientific heritage is impressive. The works of V.A. Dykhta, including the works with the co-authors, cover:

- various classes of optimal control problems (with concentrated parameters, just continuous, — including degenerate (in many publications, beginning with [22, 23]), non-smooth [41, 43, 86], — discrete [85, 90], discrete-continuous [66], as well as with distributed parameters [12, 28, 30, 54]);
- problems' transformations (including the nonlinear Goh transformation developed by Vladimir Aleksandrovich [14, 25]);
- various necessary and sufficient conditions for optimality (including the developed by Vladimir Aleksandrovich variational maximum principle [3, 33, 35, 49], positional minimum principle [82, 83, 88–90, 92] which are important, in particular, from the point of view of the Pontryagin maximum principle's strengthening in regular problems, studies of degenerate problems);
- methods for solving optimal control problems (for example, [39, 44] are for a constructing the methods for solving optimal impulse control problems based on the variational maximum principle);
- numerous analytical examples, including the investigation examples (in [14, 42, 46], etc.) for the optimal control problems on modeling a robotic manipulator, quantum systems, (ecological-)economic processes, etc., with the application and discussion of the conditions for optimality, problems' transformations, taking into account the issues of the constructions' mathematical correctness;
- researches in numerical experiments (in this regard, for example, [10, § 4.1, 4.2]);
- modeling of ecological-economic systems — this certain point notes the co-authorship of V.A. Dykhta in the collective monographs [5–9].

V.A. Dykhta did an active scientific creativity until his recent passing. We note the work [91], which was presented in December 2024 at the 40th conference 'Lyapunov Readings' and addresses positional strengthenings of the V.F. Krotov's method, and the article [92] developing the positional minimum principle. The collective computer program for solving optimal control problems of a certain type using a gradient method has been registered [93].

For the numerical solution of various optimal control problems — including degenerate problems and those with terminal constraints — the creation of new algorithms and computer programs based on the fundamental results of V.A. Dykhta and from the co-authored works, including, for example, the positional minimum principle, seems promising. For example, it is of interest to take into account the results on impulse control which were obtained for some quantum model optimization problems and presented in [14, § 3.7, 6.8], [42, 46] (V.A. Dykhta, et al.).

One of the goals of this article is to draw its readers' attention to both the scientific heritage of V.A. Dykhta and the scientific heritage of V.I. Gurman, V.F. Krotov, etc., and more broadly, to the development of the theory of optimal control. Young readers unfamiliar with the works of V.A. Dykhta are recommended to start with the video recordings of the talks of Vladimir Alexandrovich [94, 95].

This article was written taking into account the discussions with colleagues about its preliminary versions.

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