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**LINEAR SYSTEMS**

# Anisotropy–Based Estimation Design for Linear Discrete Time–Invariant Systems with Measurement Dropouts

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**Abstract**—This paper considers a time-invariant system with a set of imperfect measurements, the useful component of which may be unavailable at certain times. Virtual objects duplicating the state of the original object are introduced, and an output estimate is constructed for each of them. Using vectorization, an extended model of the system is derived in the form of a linear discrete time-invariant system with multiplicative noises. The external disturbance is chosen from the class of sequences of random vectors with a bounded level of mean anisotropy. For the system describing the estimation error dynamics, conditions for the boundedness of the anisotropic norm are obtained, under which the chosen type of estimator exists. An invertible linearizing change of variables is indicated, which allows reducing the problem of finding the estimator matrices to checking the solvability of a special system of linear matrix inequalities with a convex constraint.

*Keywords*: anisotropy-based theory, systems with multiplicative noise, sensor network with dropouts, convex optimization

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## 1. INTRODUCTION

The problems of external disturbances rejection considered in control theory were formulated back in the first half of the last century [1] and gradually transitioned from deterministic [2] and bounded disturbances [3] to stochastic ones [4, 5], interest in which persists to this day [6]. It is worth noting that the most frequently described classes of disturbances are Gaussian disturbances [7] or square-integrable (summable in the case of discrete time) [8]. The disturbance rejection problems associated with these classes, with an $\mathcal{H}_2$- or $\mathcal{H}_\infty$-optimal criterion, have both advantages and disadvantages. On the one hand, with known stochastic characteristics of the disturbance, $\mathcal{H}_2$-optimal estimators and controllers have proven themselves well; however, solutions to such problems do not possess the robustness properties inherent in $\mathcal{H}_\infty$-optimal control theory. On the other hand, $\mathcal{H}_\infty$-optimal controllers introduce excessive conservatism into the dynamics of a system with weakly colored disturbances at the input.

Anisotropy-based control theory proposes to view the problem of suppressing the influence of an external disturbance as a stochastic modification of $\mathcal{H}_\infty$-optimal theory, since the external disturbance is chosen from the class of random, rather than square-summable, sequences [9–11]. This expands the scope of anisotropy-based theory compared to $\mathcal{H}_\infty$-theory, since for deterministic and norm-bounded disturbances, the mean anisotropy level will tend to infinity, and the limiting value of the anisotropic norm will tend to the $\mathcal{H}_\infty$-norm. For random external disturbances, anisotropy-based theory will give results between $\mathcal{H}_2$- and $\mathcal{H}_\infty$-theories in terms of the cost function, depending on the value of the information criterion. The results obtained over the thirty years of existence of anisotropy-based theory cover the problems of robust stability analysis of time-invariant and time-varying systems, control synthesis, and filtering [11–13]. Later, works appeared devoted to systems whose state cannot be described by deterministic equations [14–16]. In particular, systems with multiplicative noises are a convenient form of describing the dynamics for systems representing mechanical, biological, ecological, financial, and hybrid objects [17]. Within the framework of this work, the problem of constructing an output estimate for a system with dropouts will be considered. An imperfect measurement is understood as such a measurable output of the system that, at an arbitrary time, may not contain useful information about the object of observation, but may be completely random. The case of random measurement is called dropout. It turns out that a system with one or several (possibly duplicating each other) imperfect measurable outputs can be described by difference equations with multiplicative noises, for which some results for time-varying systems have already been obtained in anisotropy-based theory.

Anisotropy-based theory allows combining the features of both $\mathcal{H}_2$- and $H_\infty$-optimal control theories, and in limiting cases even provide identical results, being in this respect a generalizing theory. Therefore, when constructing an output estimate or generating control, it makes sense to choose the anisotropy-based approach in the case where the stochastic characteristics of the external disturbance are not specified. If the expectation and covariance matrix of the disturbance are known, then the solution obtained within the framework of anisotropy-based theory is generally less conservative, since it is not designed for the so-called "worst case" of disturbance.

The paper is organized as follows: Section 2 contains preliminary information—the necessary minimum of concepts from anisotropy-based theory; Section 3 presents the problem statement; Section 4 derives the main result; Section 5 presents simulation results; Section 6 provides a brief conclusion.

## 2. PRELIMINARY INFORMATION

We list the main definitions from anisotropy-based theory that directly relate to the problem under consideration. More detailed material can be found in [18, 19].

### *2.1. Anisotropy of a Random Vector*

**Definition 1** [19]. The anisotropy of a random vector $w$ with values in $\mathbb{R}^m$ is the value equal to

$$\mathbf{A}(w) = \min_{\lambda>0} \mathbf{D}(f||p_{m,\lambda}) = \frac{m}{2} \ln\left(\frac{2\pi e}{m}\mathbf{E}[|w|^2]\right) - h(w),$$

where $f$ is the probability density function of the vector $w$ with respect to the Lebesgue measure in $\mathbb{R}^m$, $h(w) = -\mathbf{E}[\ln f(w)]$ is the differential entropy, $\lambda$ is a positive parameter defining the density

$$p_{m,\lambda}(x) = (2\pi\lambda)^{-m/2} \exp\left(-\frac{|x|^2}{2\lambda}\right), \quad x \in \mathbb{R}^m,$$

of the isotropic Gaussian distribution in $\mathbb{R}^m$ with zero mean and scalar covariance matrix $\lambda I_m$, where $I_m$ is the identity matrix of order $m$, and $\mathbf{E}[\cdot]$ is the expectation operator.

This definition is related to relative entropy and characterizes the minimum measure of difference between a random vector and the set of centered Gaussian vectors with a scalar covariance matrix.

### *2.2. Mean Anisotropy of a Sequence of Random Vectors*

**Definition 2** [9]. For a stationary ergodic sequence $W = \{w_k\}$ of Gaussian vectors, the concept of mean anisotropy is introduced, equal to the value

$$\overline{\mathbf{A}}(W) = \lim_{N\to\infty} \frac{\mathbf{A}(W_{0:N-1})}{N},$$

where $W_{0:N-1} = [w_0^{\mathrm{T}}, \ldots, w_{N-1}^{\mathrm{T}}]^{\mathrm{T}}$ is the extended vector.

The mean anisotropy level is used to describe the properties of an external disturbance within the framework of uncertainty in stochastic parameters when analyzing the robust stability of linear discrete time-invariant systems, since the extended vector of any distribution different from the reference one a centered Gaussian with a scalar covariance matrix will have an increasing anisotropy as the time horizon increases.

### *2.3. Anisotropic Norm of a Time-Invariant System*

Consider a linear system $F$ with input $W \in \mathcal{L}_2^m$ and output $Z \in \mathcal{L}_2^p$, where $\mathcal{L}_2^*$ denotes the Lebesgue space of vectors with finite norm. The sequence $W$ is generated by a shaping filter $G$ from standard Gaussian white noise $V$:

$$w_j = \sum_{k=0}^{\infty} g_k v_{j-k}, \quad j \in \mathbb{Z},$$

where $g_k \in \mathbb{R}^{m\times m}$. Let $G(z)$ denote the transfer function of the former filter:

$$G(z) = \sum_{k=0}^{\infty} g_k z^k, \quad |z| < 1, \quad z \in \mathbb{C}.$$

Let the set of former filters generating sequences of random vectors with bounded mean anisotropy be denoted as follows [20]:

$$\mathcal{G}_a = \{G \in \mathcal{H}_2^{m\times m} : W = GV,\ \overline{\mathbf{A}}(W) \leqslant a\},$$

where $V = \{v_k\}_{k\in\mathbb{Z}}$ is standard Gaussian white noise with zero mean and unit covariance matrix.

**Definition 3** [10]. The anisotropic norm $|\!|\!|F|\!|\!|_a$ of a linear system $F$ is the quantity

$$|\!|\!|F|\!|\!|_a = \sup_{G\in\mathcal{G}_a} \frac{\|FG\|_2}{\|G\|_2},$$

where $\|G\|_2 = \left(\sum_{k=0}^{\infty} \mathrm{tr}(g_k g_k^{\mathrm{T}})\right)^{1/2}$ is the $\mathcal{H}_2$-norm of the transfer function $G(z)$.

## 3. PROBLEM STATEMENT

Consider a linear discrete time-invariant system with a set of imperfect sensors

$$\begin{aligned} x_{k+1} &= Ax_k + B_w w_k, \\ z_k &= C_z x_k + D_z w_k, \\ y_{j,k} &= \lambda_{j,k} C_j x_k + D_j w_k, \quad j = \overline{1,n}, \end{aligned} \tag{1}$$

where

$$P(\lambda_{j,k} = 1) = p_j, \quad j = \overline{1,n},$$

characterize the probability that the current measurement $y_{j,k}$ contains information about the state vector. In the case $\lambda_{j,k} = 1$, the measured output contains useful data; otherwise, $\lambda_{j,k} = 0$, it contains only the disturbance vector $D_j w_k$. The matrices $A, B_w, C_z, D_z, C_j, D_j,\ j = \overline{1,n}$, of system (1) are known and have dimensions consistent with the vectors $x_k,\ w_k,\ z_k,\ y_{j,k}$. The external disturbance is a stationary Gaussian sequence of random vectors $W = \{w_k\}_{k\geqslant 0}$, for which a constraint on the mean anisotropy level of the entire sequence is known: $\overline{\mathbf{A}}(W) \leqslant a,\ a \geqslant 0$. It is assumed that all elements of the sequence $W$ and the random variables $\lambda_{j,k}$ are mutually independent.

*Problem 1.* For the chosen estimator model

$$\begin{aligned} \hat{x}_{j,k+1} &= \sum_{i=1}^{n} \mathfrak{a}_{ji} \left( A\hat{x}_{i,k} + H_{ji}(y_{i,k} - \hat{y}_{i,k}) \right), \\ \hat{z}_{j,k} &= \sum_{i=1}^{n} \mathfrak{a}_{ji} C_z \hat{x}_{i,k}, \\ \hat{y}_{j,k} &= p_j C_j \hat{x}_{j,k}, \quad j = \overline{1,n}, \end{aligned} \tag{2}$$

where $\mathfrak{a} \in \mathbb{R}^{n\times n}$ is the adjacency matrix characterizing the information exchange between sensors, it is required to find such values of the parameters $H_{ji}$ for which the anisotropic norm of the system $F$ from the external disturbance to the estimation error vector is bounded by a minimum value $\gamma$:

$$|\!|\!| F |\!|\!|_a \leqslant \gamma.$$

It should be noted here that the adjacency matrix $\mathfrak{a}$ is not necessarily symmetric. This matrix describes a directed graph whose vertices are the sensors (detectors/sensors or any available measurements), and the directed edges correspond to data exchange between the vertices connected by these edges. As a result, the estimate can be constructed not based on each individual sensor, but by collecting a consensus variant of the estimate, where the contribution of each sensor is accounted for using a correction coefficient of the matrix $\mathfrak{a}$. Thus, the coefficients of the adjacency matrix have a simple physical meaning, which lies in the level of "trust" in the information coming from other sensors to the one forming the estimate. A natural requirement is the element-wise non-negativity and the equality to one of the sum of elements in each row:

$$\sum_{i=1}^{n} \mathfrak{a}_{ji} = 1, \quad \forall j = \overline{1,n}.$$

## 4. MAIN RESULT

This section provides a solution to Problem 1. The idea is to transform the object model into the form of a system with multiplicative noises and then check a sufficient condition for the boundedness of the anisotropic norm of such a system in terms of solving a convex optimization problem.

### *4.1. Construction of Models in Terms of Estimation Errors*

At the first step, we define virtual models as follows: for each existing estimated output $y_{j,k}$, we introduce a state $x_{j,k} \in \mathbb{R}^{n_x}$, duplicating the state $x_k$ of the object (1):

$$\begin{aligned} x_{j,k+1} &= Ax_{j,k} + B_w w_k, \\ z_{j,k} &= C_z x_{j,k} + D_z w_k, \\ y_{j,k} &= \lambda_{j,k} C_j x_{j,k} + D_j w_k, \end{aligned} \tag{3}$$

where $j = \overline{1,n}$. In models of the form (1), each object has only one measured output. For the states $x_{j,k}$ and outputs $z_{j,k}$, we introduce error vectors

$$\tilde{x}_{j,k} = x_{j,k} - \hat{x}_{j,k},$$
$$\tilde{z}_{j,k} = z_{j,k} - \hat{z}_{j,k},$$

which will have the following dynamics equations:

$$\begin{aligned}\tilde{x}_{j,k+1} &= Ax_{j,k} - \sum_{i=1}^{n} \mathfrak{a}_{ji} A\hat{x}_{i,k} + \left(B_w - \sum_{i=1}^{n} \mathfrak{a}_{ji} H_{ji} D_i\right) w_k \\ &\quad - \sum_{i=1}^{n} \mathfrak{a}_{ji} H_{ji}(\lambda_{i,k} - p_i) C_i x_{i,k} + \sum_{i=1}^{n} \mathfrak{a}_{ji}(A - p_i H_{ji} C_i)\tilde{x}_{i,k}, \\ \tilde{z}_{j,k} &= C_z \left(x_{j,k} - \sum_{i=1}^{n} \mathfrak{a}_{ji} x_{i,k}\right) + \sum_{i=1}^{n} \mathfrak{a}_{ji} C_z \tilde{x}_{i,k} + D_z w_k.\end{aligned}$$

### *4.2. Transformation of the Model to a System with Multiplicative Noises*

Let us compose extended vectors of state, state errors, and estimates from all vectors $x_{j,k}$, $\tilde{x}_{j,k}$, $\tilde{z}_{j,k}$, $j = \overline{1,n}$:

$$\overline{x}_k = (x_{1,k}^{\mathrm{T}}, \ldots, x_{n,k}^{\mathrm{T}})^{\mathrm{T}},$$
$$\tilde{x}_k = (\tilde{x}_{1,k}^{\mathrm{T}}, \ldots, \tilde{x}_{n,k}^{\mathrm{T}})^{\mathrm{T}},$$
$$\tilde{z}_k = (\tilde{z}_{1,k}^{\mathrm{T}}, \ldots, \tilde{z}_{n,k}^{\mathrm{T}})^{\mathrm{T}}.$$

Define the combined state from the vectors $\overline{x}_k$ and $\tilde{x}_k$:

$$\zeta_k = (\overline{x}_k^{\mathrm{T}}, \tilde{x}_k^{\mathrm{T}})^{\mathrm{T}}.$$

Now it is possible to write the dynamics equation in terms of errors depending on the external disturbance:

$$\begin{aligned}\zeta_{k+1} &= \left(\mathcal{A}_0 + \sum_{i=1}^{n} \xi_{i,k} \mathcal{A}_i\right) \zeta_k + \mathcal{B} w_k, \\ \tilde{z}_k &= \mathcal{M}\zeta_k + \mathcal{N} w_k,\end{aligned} \tag{4}$$

where the following notations are adopted:

$$\xi_{i,k} = \lambda_{i,k} - p_i, \quad i = \overline{1,n},$$

and the matrices included in expressions (4) have the form:

$$\mathcal{A}_0 = \begin{bmatrix} \overline{A} & 0_{nn_x \times nn_x} \\ \overline{A} - \overline{A}^{\mathfrak{a}} & \overline{A}^{\mathfrak{a}} - \overline{H}^{\mathfrak{a}} \overline{C}^{p} \end{bmatrix}, \quad \mathcal{A}_i = \begin{bmatrix} 0_{nn_x \times nn_x} & 0_{nn_x \times nn_x} \\ \overline{H}^{\mathfrak{a}} \overline{C}_i & 0_{nn_x \times nn_x} \end{bmatrix},$$
$$\mathcal{B} = \begin{bmatrix} \overline{B} \\ \overline{B} - \overline{H}^{\mathfrak{a}} \overline{D} \end{bmatrix},$$
$$\mathcal{M} = \begin{bmatrix} \overline{M} - \overline{M}^{\mathfrak{a}} & \overline{M}^{\mathfrak{a}} \end{bmatrix}, \quad \mathcal{N} = \overline{N},$$

with notations

$$\begin{aligned}
\overline{A} &= I_n \otimes A, & \overline{A}^{\mathfrak{a}} &= \mathfrak{a} \otimes A, \\
\overline{B} &= \mathrm{col}(n) \otimes B, & \overline{H}^{\mathfrak{a}} &= \underset{j,i=\overline{1,n}}{\mathrm{block}}(\mathfrak{a}_{ji} H_{ji}), \\
\overline{C}^{p} &= \underset{i=\overline{1,n}}{\mathrm{diag}}(p_i C_i), & \overline{C}_j &= \underset{i=\overline{1,n}}{\mathrm{diag}}(\delta_{ij} C_i), \\
\overline{D} &= \underset{j=\overline{1,n}}{\mathrm{col}}(D_j), & \overline{M} &= I_n \otimes C_z, \\
\overline{M}^{\mathfrak{a}} &= \mathfrak{a} \otimes C_z, & \overline{N} &= \mathrm{col}(n) \otimes D_z,
\end{aligned}$$

where $\mathrm{col}(n)$ denotes the column vector of length $n$ consisting of ones; $\underset{j=\overline{1,n}}{\mathrm{col}}(X_j)$ is the matrix obtained by arranging the matrices $X_j$, $j = \overline{1,n}$, into a column; the matrix $\underset{j,i=\overline{1,n}}{\mathrm{block}}(X_{ji})$ is a block matrix consisting of the corresponding blocks; $\underset{i=\overline{1,n}}{\mathrm{diag}}(X_i)$ is a diagonal matrix with the matrices $X_i$, $i = \overline{1,n}$, as blocks on the main diagonal, and all off-diagonal blocks are zero; $\otimes$ denotes the Kronecker product.

### *4.3. Condition for Boundedness of the Anisotropic Norm*

The following theorem provides sufficient conditions for the boundedness of the anisotropic norm for system (4).

**Theorem 1** [15]. *For a system $F$ of the form* (4) *with additional condition*

$$\lim_{k\to\infty} \rho\left(\left(E\left[\mathcal{A}^k\right]\right)^{\frac{1}{k}}\right) < 1,$$

*where $\mathcal{A} = \mathcal{A}_0 + \sum\limits_{i=1}^{n} \xi_{i,k}\mathcal{A}_i$ given a known constraint $a$ on the mean anisotropy level of the external disturbance sequence $\{w_k\}$, the anisotropic norm will be bounded by $\gamma$ if there exists a solution with respect to the matrices $R$, $S$ and the scalar parameter $q \in \left[0, \|F\|_\infty^{-2}\right)$, $\sigma_i = p_i(1-p_i)$ to the system of inequalities*

$$R \succ \sum_{i=0}^{n} \sigma_i^2 \mathcal{A}_i^{\mathrm{T}} R \mathcal{A}_i + q\mathcal{M}^{\mathrm{T}}\mathcal{M} + L^{\mathrm{T}} S^{-1} L, \tag{5}$$

$$S = (I_{m_w} - q\mathcal{N}^{\mathrm{T}}\mathcal{N} - \mathcal{B}^{\mathrm{T}} R \mathcal{B})^{-1}, \tag{6}$$

$$L = S(\mathcal{B}^{\mathrm{T}} R \mathcal{A} + q\mathcal{N}^{\top}\mathcal{M})$$

*subject to the convex constraint*

$$-\frac{1}{2}\ln\det\left((1-q\gamma^2)S\right) \geqslant a, \tag{7}$$

*there the expression $X \succ 0$ should be understood in the sense of positive definiteness of the matrix $X$.*

Theorem 1 contains sufficient conditions for the boundedness of the anisotropic norm of a system with multiplicative noises (4) in the form of inequalities. However, inequality (5) is nonlinear due to the dependence of the matrices $\mathcal{B}, \mathcal{A}_i, i = \overline{1,n}$, on the estimator matrices $H_{ji}, j, i = \overline{1,n}$, and their multiplication by the matrix $R$ of the solution of this inequality. To get away from this problem and remain within the constraints in the form of linear matrix inequalities, we formulate the solution to the problem as conditions for the solvability of a system of inequalities of a special form.

**Theorem 2.** *Suppose that for system* (1) *a constraint a is known on the mean anisotropy level of the disturbance sequence* $W = \{w_k\}_{k\geqslant 0}$. *An estimator of the form* (2) *will guarantee an upper bound by* $\gamma$ *for the anisotropic norm of the estimation error system if the following system of inequalities*

$$\begin{bmatrix} R-\mathcal{M}^{\mathrm{T}}\mathcal{M} & * & * & * & * & \cdots & * \\ \mathcal{N}^{\mathrm{T}}\mathcal{M} & \eta I_{m_w}-\mathcal{N}^{\mathrm{T}}\mathcal{N} & * & * & * & \cdots & * \\ R\mathcal{A}_{00}+X\mathcal{A}_{01} & -R\mathcal{B}_{00}-X\mathcal{B}_{01} & R & * & * & \cdots & * \\ \sigma_1 X\mathcal{A}_{11} & 0 & 0 & R & * & \cdots & * \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_n X\mathcal{A}_{1n} & 0 & 0 & 0 & 0 & \cdots & R \end{bmatrix} \succ 0, \tag{8}$$

$$\begin{bmatrix} \eta I_{m_w}-\Psi-\mathcal{N}^{\mathrm{T}}\mathcal{N} & * \\ R\mathcal{B}_{00}+X\mathcal{B}_{01} & R \end{bmatrix} \succ 0, \tag{9}$$

$$\ln\det\Psi \geqslant 2a + m_w\ln(\eta-\gamma^2), \tag{10}$$

*has a solution with respect to the matrices* $R = R^{\mathrm{T}} \succ 0$, $X \in \mathbb{R}^{4nn_x\times 2np_y}$, *and the scalar parameter* $\eta > 0$, *where* $\sigma_i = p_i(1-p_i)$,

$$\mathcal{A}_{00} = \begin{bmatrix} \overline{A} & 0 \\ \overline{A}-\overline{A}^{\mathfrak{a}} & \overline{A}^{\mathfrak{a}} \end{bmatrix}, \quad \mathcal{A}_{01} = \begin{bmatrix} 0 & 0 \\ 0 & -\overline{C}^{p} \end{bmatrix},$$

$$\mathcal{A}_{j1} = \begin{bmatrix} 0 & 0 \\ -\overline{C}_j & 0 \end{bmatrix}, \quad j=\overline{1,n},$$

$$\mathcal{B}_{00} = \begin{bmatrix} \overline{B} \\ \overline{B} \end{bmatrix}, \ \mathcal{B}_{01} = \begin{bmatrix} 0 \\ -\overline{N} \end{bmatrix}.$$

The proof of Theorem 2 is given in Appendix.

It should be mentioned here that among the entire set of solution matrices $R$ of system (8)–(10), we will choose only block-diagonal ones, since this will be necessary for the numerical solution of the problem using standard tools.

### *4.4. Calculation of the Estimator Matrix Parameters*

In the event of a successful solution of the matrix inequalities (8)–(10), the set of estimator matrices $H_{ji}, j, i = \overline{1,n}$, can be calculated according to the inverse substitution

$$\overline{H}^{\mathfrak{a}} = [0_{2nn_x\times 2nn_x} \ \ I_{2nn_x}]R^{-1}X \begin{bmatrix} 0_{2np_y\times 2np_y} \\ I_{2np_y} \end{bmatrix}. \tag{11}$$

The problem of finding the minimum value of the anisotropic norm bound $\gamma$ can be formulated as a convex optimization problem:

$$\gamma^2 \underset{(8)-(10),\mathcal{R},\Psi,\eta,\gamma^2}{\longrightarrow} \min. \tag{12}$$

Direct calculation of the matrices $H_{ji}, j, i = \overline{1,n}$, is not necessary, since the spatial implementation of the estimator (2) depends on the blocks of matrix (11). The solution of the convex optimization problem (12) will correspond to the minimum constraint on the anisotropic norm of the estimation error system.

## 5. SIMULATION

As an example, consider a model consisting of a quadrotor aerial vehicle fixed on a gyroscopic stand. The object can rotate freely around three axes. It is necessary to estimate its roll angle using the gyroscope and camera available on the object, located frontally in front of the object. We assume that the object is closed by stabilizing control based on an LQG controller. The discrete model of such a system, according to the dynamics equations (1), in the state space has the following form:

$$A = \begin{bmatrix} 0.9988 & 0.0000 & 0.0000 & 0.0095 & 0.0000 & 0.0000 \\ 0.0000 & 0.9999 & 0.0000 & 0.0000 & 0.0099 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0100 \\ -0.2411 & 0.0000 & 0.0000 & 0.8926 & 0.0000 & 0.0000 \\ 0.0000 & -0.0141 & 0.0000 & 0.0000 & 0.9763 & 0.0000 \\ 0.0000 & 0.0000 & -0.0009 & 0.0000 & 0.0000 & 0.9957 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 1.0028 & 0.0000 & 0.0000 \\ 0.0000 & 0.0027 & 0.0000 \\ 0.0000 & 0.0000 & 0.0008 \end{bmatrix},$$

$$C_z = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad D_z = 0, \quad C_1 = \begin{bmatrix} I_3 & 0_{3\times 3} \end{bmatrix}, \quad C_2 = C_z, \quad D_1 = D_2 = 0.05 \cdot I_3,$$

and parameters $p_1 = 0.9$, $p_2 = 0.95$. The first measured output corresponds to data from the onboard gyroscope, the second from the frontal camera. It is assumed that the angle values are recovered from the angular velocity measurements by integration; at the initial time, the estimates are considered zero.

Figure 1 shows the simulation results for the mean anisotropy level equal to 30. The solid line corresponds to the estimated output. Estimates 1 and 2 correspond to the case where the readings from the other measured output were used with a weighting coefficient of 0.2; in this case, the

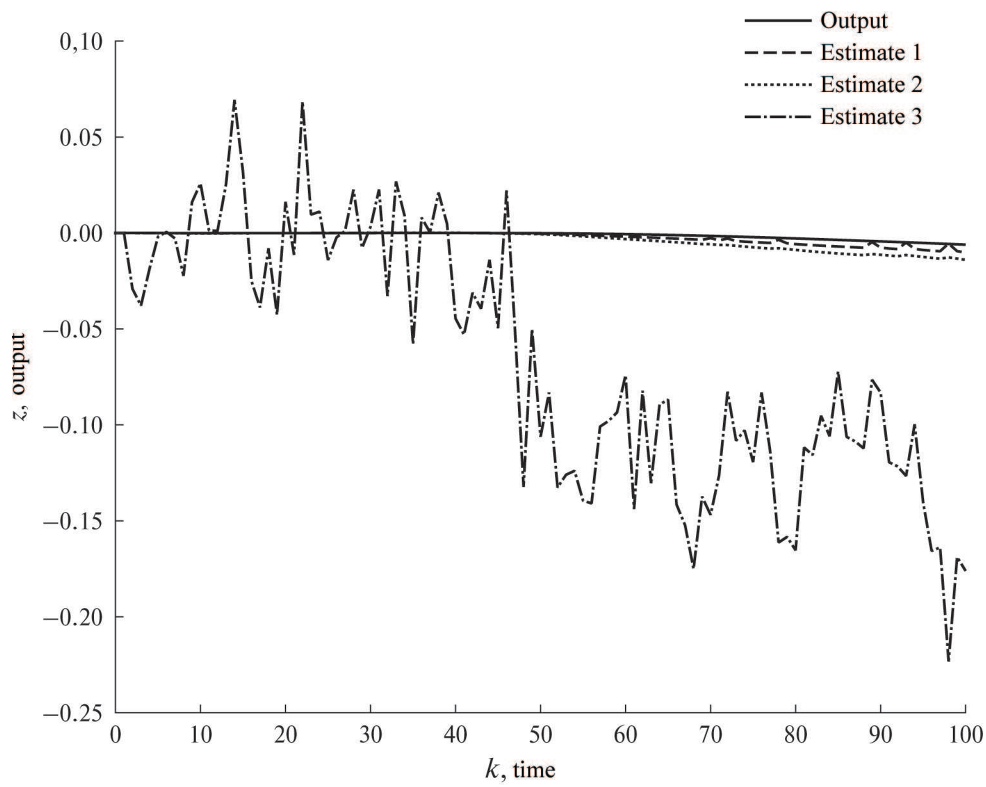


**Fig. 1.** Estimated output.

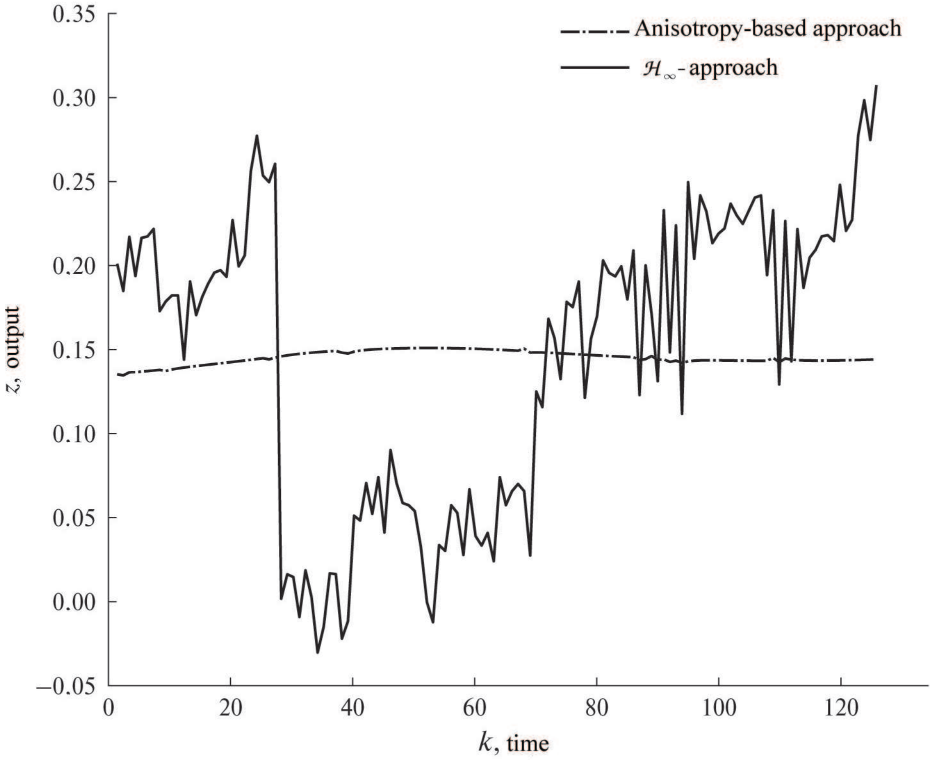


**Fig. 2.** Comparison of estimations.

adjacency matrix has the following form:

$$\mathfrak{a} = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}.$$

Estimate 3 corresponds to the case where only the first measured output was used to construct the estimate.

For comparison, we present the values of the Euclidean norms of the estimated output $\|z\| = 0.0221$ and the three obtained estimates: $\|\hat{z}_1\| = 0.0385$, $\|\hat{z}_2\| = 0.0598$, $\|\hat{z}_3\| = 1.0222$.

Figure 2 presents the simulation results under the influence of an external disturbance with a fixed mean anisotropy level equal to 30. In this case, we compare only the consensus estimate obtained based on two measurements. The dash-dotted line denotes the estimate of the roll angle based on the estimator using the anisotropy-based approach, the solid line denotes the $\mathcal{H}_\infty$-optimal estimate. Although the values of the Euclidean norms of the anisotropy-based estimate $\|\hat{z}_{AB}\| = 1.6308$ and the $\mathcal{H}_\infty$ estimate $\|\hat{z}_{\mathcal{H}_\infty}\| = 1.9165$ differ by 15%, it is noticeable from the graphs that in the considered example with imperfect measurements, the nature of the obtained estimates differs significantly, which indicates the advantage of using the anisotropy-based estimator in this case.

## 6. CONCLUSION

The paper solves the problem of constructing an estimate for a system whose measured outputs may be unavailable at an arbitrary time. To this end, it is proposed to duplicate such outputs and build a consensus estimate for the output of the object of observation. Since the solution to the problem can be obtained using standard application software packages for computations, it seems possible to use the synthesized estimator, for example, for constructing output feedback control for dynamic objects.

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*APPENDIX*

## A.1. PROOF OF THEOREM 2

Apply the Schur complement lemma [21] to inequality (5), which leads to the appearance of an inequality of the following form:

$$\begin{bmatrix} R-\mathcal{M}^{\mathrm{T}}\mathcal{M} & * & * & * & * & \cdots & * \\ \mathcal{N}^{\mathrm{T}}\mathcal{M} & \eta I_{m_w}-\mathcal{N}^{\mathrm{T}}\mathcal{N} & * & * & * & \cdots & * \\ R\mathcal{A}_0 & -R\mathcal{B} & R & * & * & \cdots & * \\ \sigma_1 R\mathcal{A}_1 & 0 & 0 & R & * & \cdots & * \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_n R\mathcal{A}_n & 0 & 0 & 0 & 0 & \cdots & R \end{bmatrix} \succ 0,$$

where the substitution $R = q^{-1}R$ is taken into account. As can be seen, the last inequality is still nonlinear. To reduce it to a linear form, we introduce the following change of variables:

$$X = RU = R\begin{bmatrix} Y & 0 \\ 0 & \overline{H}^{\mathfrak{a}} \end{bmatrix},$$

where $Y$ is some matrix (it is worth noting in advance that this matrix does not affect the solution of the problem, namely, the values of the estimator matrices $H_{ji}, j, i = \overline{1,n}$). Taking into account the last change of matrix variables, we obtain inequality (8).

To derive inequality (9), we define an additional positive definite matrix $\Psi$ related to the matrix $S$ as follows:

$$0 \prec \frac{1}{\eta}\Psi \prec S^{-1},$$

where $\eta^{-1} = q$. Taking into account the positive definiteness of the matrix $\Psi$, the change of variables $X = RU$, and applying the Schur complement lemma, we obtain inequality (9). The convex constraint (10) follows from inequality (7) after the corresponding changes of variables. The theorem is proved.

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